



Research Institute for Mathematical Sciences - Kyoto University

# HOMOTOPICAL ANABELIAN GEOMETRY

Reading Seminar - Arithmetic Geometry

Second Semester 2019-2020

京都大学

Organizer: Benjamin Collas, RIMS - Kyoto University.

The goal of this seminar is to present some recent anabelian results which ensue the introduction of Artin-Mazur and Friedlander *homotopical étale context* in arithmetic-geometry by A. Schmidt and J. Stix, with in particular

*The existence in smooth varieties over number field of an anabelian Artin neighbourhood,*

which is a result that was predicted by Grothendieck in his letter to Faltings.

We introduce the required context and properties of Artin-Mazur étale homotopy type in terms of Quillen model category, present Tamagawa's property of points separation and Mochizuki's Theorem A, and establish the aforementioned Theorem with Hoshi's generalization to sub- $p$ -adic field for polycurves. A special attention is in particular given on *how classical and homotopical anabelian geometry interact together* within the topics of **(1) Homotopical Algebraic Geometry**, **(2) Anabelian Geometry in Higher dimensions**, and **(3) Anabelian Zariski Neighbourhoods**. The use of homotopical methods opens some new perspective in arithmetic geometry, stacks and motivic theory.

## Talk 1

### Étale Homotopy Type

Artin-Mazur & Friedlander étale homotopy and topology type for schemes, homotopy pro-groups and completion functor, comparison theorem; étale cohomology via hypercoverings.

## Talk 2

### Model Category for Pro-Spaces

Principle of homotopy category, Isaksen model category for pro-spaces & comparison to Artin-Mazur construction; Postnikov towers.

## Talk 3

### Homotopic Algebraic Geometry

Algebraic properties of pro-spaces: covering base changes, étale cohomology as Galois modules of étale homotopy types, homotopy groups of pro-spaces and topology.

## Talk 4

### (Un)Pointed Pro-Spaces

A key points of topological type: morphisms in homotopy categories of pointed and unpointed pro-spaces for strongly center-free Galois groups; Monodromy action in étale homotopy types.

## Talk 5

### A Homotopical Theorem A

A sketch of proof of Mochizuki's Theorem over sub- $p$ -adic fields, homotopy reformulation of Theorem A with respect to étale  $K(\pi, 1)$  properties.

## Talk 6

### A Weak Anabelian Result

Over number fields, existence of the functorial retraction  $(-)_et: \text{Isom}_k(X, Y) \rightleftarrows \text{Hom}_{\text{Ho}(\text{pro-Sp}) \downarrow k_{et}}(X_{et}, Y_{et})$ :  $r$ , via Lefschetz trace formula and Tamagawa's separating point Lemma.

## Talk 7

### Retraction & Class Preservation

Kernel of the retraction and  $\pi_1^{et}(X)$  class-preservation as trivial Tate-type, via Čebatorev density Theorem and the use of regular models.

## Talk 8

### Anabelian Pro- $K(\pi, 1)$

Existence of  $K(\pi, 1)$  in pro-spaces for pro-groups, étale and topological consequences of Artin good neighbourhood and Serre "bonté".

## Talk 9

### Anabelian Neighbourhoods

Proof of the main theorem via properties of the kernel retraction, via Theorem A & existence of good Artin neighbourhoods.

## Talk 10

### Relative Anabelian Neighbourhood & Polycurves

Hoshi's extension from number field to sub- $p$ -field of the strong anabelian results for polycurves.



The **Homotopical Anabelian Geometry Seminar** takes place every two weeks at the RIMS and is addressed to Master students, PhD and postdoctoral researchers interested in classical and homotopical anabelian geometry. It is expected for speakers to illustrate fundamental notions via geometric motivations and key examples. Pleaser refer to the website for schedule and detailed programme; contact the organizer for attending or giving a talk.